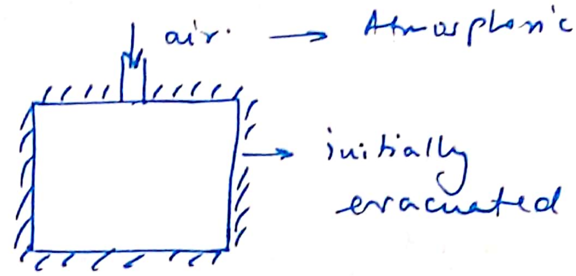
Tutorial-4 11-02-2020

1.)



→ Atmospheric air at 95 kPa and $17^\circ C = 290 K$.

Atmospheric air is at 95 kPa and $17^\circ C = 290 K$.

Air starts entering the initially evacuated tank, and continues to enter until the tank pressure reaches 95 kPa.

Question :- Find the final temperature of the air in the tank.

Mass balance :-

$$m_{in} - m_{out} = m_{final} - m_{initial} \quad \text{--- (1)}$$

No mass flows out of the tank, so $m_{out} = 0$.

No mass is initially present in the system, so

$m_{initial} = 0$

So, eqn (1) reduces to :-

$$m_{in} = m_{final} \quad \text{- (2)}.$$

Energy balance :-

$$Q_{in} + W_{in} + m_{in}\left[h_{in} + KE_{in} + PE_{in}\right] - \left[Q_{out} + W_{out} + m_{out}\{h_{out} + KE_{out} + PE_{out}\}\right] =$$

$$m_{final}\left[u_{final} + KE_{final} + PE_{final}\right] - m_{initial}\left[u_{initial} + PE_{initial} + KE_{initial}\right] \quad \text{--- (3)}.$$

In eqn (3),

(i) K.E. and P.E. are ignored in all the terms.

(ii) $m_{initial} = 0$

(iii) $m_{out} = 0$

(iv) Since the tank is rigid, $W_{in} = W_{out} = 0$

(v) Since the tank is insulated, $Q_{in} = Q_{out} = 0$

Thus,

$$\cancelto{0}{Q_{in}} + \cancelto{0}{W_{in}} + m_{in}\left[h_{in} + \cancelto{0}{KE_{in}} + \cancelto{0}{PE_{in}}\right] -$$

$$\left[\cancelto{0}{Q_{out}} + \cancelto{0}{W_{out}} + \cancelto{0}{m_{out}}\left\{h_{out} + \cancelto{0}{KE_{out}} + \cancelto{0}{PE_{out}}\right\}\right] =$$

$$m_f\left[u_f + \cancelto{0}{KE_f} + \cancelto{0}{PE_f}\right] - \cancelto{0}{m_{initial,i}}\left[u_i + \cancelto{0}{KE_i} + \cancelto{0}{PE_i}\right]$$

$$\Rightarrow m_{in} h_{in} = m_f u_f \quad \text{— (4)}$$

And, from eqn (2), $m_{in} = m_f$.

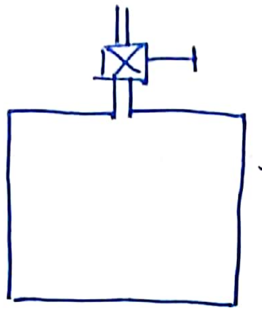
$$\therefore m_f h_{in} = m_f u_f$$

$$\Rightarrow h_{in} = u_f \Rightarrow C_p T_{in} = C_v T_{\text{air in tank}}$$

$$\Rightarrow T_{\text{air in tank}} = \frac{C_p}{C_v} T_{in} = \frac{C_p}{C_v} \cdot T_{atm}.$$

[T_{in} = Temp. of air entering the tank].

2.)



$P_{atm} = 100 kPa.$

$T_{atm} = 27°C = 300 K.$

$C_p = 1.018 kJ/kg/K$

$C_v = 0.718 kJ/kg/K$

→ 20L evacuated rigid bottle.

$20L = 0.02 m^3$

Initially, the bottle is evacuated. Then the valve is opened, and the air fills the bottle until the bottle air is in thermal & mechanical equilibrium with the surrounding atmosphere.

To find :- net heat transfer through the wall of the bottle during the filling process.

Mass balance :-

$$m_{in} - m_{out} = m_f - m_i \quad — (1)$$

f - final state of air in bottle
i = initial state of air in bottle.

$M_{out} = 0$

$M_i = 0$

$$\therefore \quad m_{in} = m_f \quad — (2)$$

Energy balance :-

$$Q_{in} + W_{in} + m_{in}[h_{in} + KE_{in} + PE_{in}] -$$

$$[Q_{out} + W_{out} + m_{out}\{h_{out} + KE_{out} + PE_{out}\}] =$$

$$m_f[u_f + KE_f + PE_f] - m_i[u_i + KE_i + PE_i]. \quad — (3)$$

Since :- (i) Tank is rigid, therefore $W_{in} = W_{out} = 0$
(ii) KE & PE interactions are ignored
(iii) $m_{out} = 0$ (iv) $m_i = 0$

Eqn (3) reduces to :-

$$Q_{in} + m_{in}h_{in} - Q_{out} = m_f u_f \quad — (4)$$

(4) $\rightarrow$ $Q_{in} + m_{in} h_{in} - Q_{out} = m_f u_f$

$\Rightarrow$ $Q_{in} - Q_{out} = m_f u_f - m_{in} h_{in}$ — (5)

From eqn (2), $m_{in} = m_f$.

$\therefore$ Eqn (5) goes to :- $Q_{in} - Q_{out} = m_f (u_f - h_{in})$ — (6)

m_f = mass of air finally present in the bottle. At this stage, the air in the bottle is in thermal & mechanical equilibrium with the atmosphere.

$\therefore$ $P_f V_f = m_f R T_f$

$\Rightarrow$ $m_f = \dfrac{P_f V_f}{R T_f} = \dfrac{100 \text{ kPa} \times 0.02 m^3}{0.287 \times 300 K}$.

$= 0.02323 \text{ kg}$ — (7).

$u_f = C_v T_f = 0.718 \dfrac{kJ}{kg K} \times 300 K = 215.4 \dfrac{kJ}{kg}$

$h_{in} = C_p T_{in} = 1.018 \dfrac{kJ}{kg K} \times 300 K = 305.4 \dfrac{kJ}{kg}$ — (8)

Substituting (7) & (8) in (6),

$Q_{in} - Q_{out} = 0.02323 \times (215.4 - 305.4)$

$= -2.0907$ kJ

$\therefore$ $Q_{in} - Q_{out} = -2.0907$ kJ

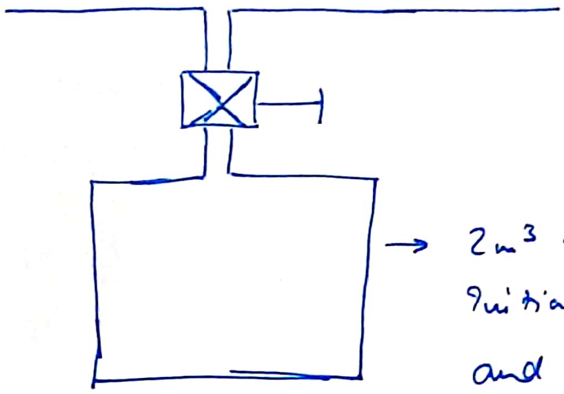
3.

Supply line with air

$\rightarrow$ 600 kPa, $22°C = 295\,K$

$C_p\,(air) = 1.018\ kJ/kg/K$

$C_v\,(air) = 0.718\ kJ/kg/K$



$\rightarrow$ $2\,m^3$ rigid tank.

Initially contains air at 100 kPa and $22°C = 295\,K$.

The valve is opened until the pressure in the tank reaches the supply line pressure, and $T_f = 77°C = 350\,K$

(i) KE, PE interactions to be ignored.

(ii) Since the tank is rigid, $W_{in} = W_{out} = 0$ — 1

(iii) Since no mass flows out of the tank, $m_{out} = 0$

Mass balance:-

$$m_{in} - \cancel{m_{out}} = m_f - m_i$$

$$\Rightarrow m_{in} = m_f - m_i = m_{final} - m_{initial} \quad -(2)$$

Energy balance:-

$$Q_{in} + \cancel{W_{in}} + m_{in}\left[h_{in} + \cancel{KE_{in}} + \cancel{PE_{in}}\right] - \left[Q_{out} + \cancel{W_{out}} + \cancel{m_{out}}\left\{h_{out} + \cancel{KE_{out}} + \cancel{PE_{out}}\right\}\right]$$

$$= m_f\left[u_f + \cancel{KE_f} + \cancel{PE_f}\right] - m_i\left[u_i + \cancel{KE_i} + \cancel{PE_i}\right] \quad -(3)$$

$$\Rightarrow Q_{in} + m_{in} h_{in} - Q_{out} = m_f u_f - m_i u_i \quad -(4).$$

To determine (a) mass of air that has entered the tank.

Look at eqn (2) $\Rightarrow$ $m_{in} = m_{final} - m_{initial}$.

P.T.O

Initial state of tank

$P_i = 100\ kPa$

$V_i = 2\ m^3$

$T_i = 22°C = 295\ K.$

$\therefore m_{initial} = \frac{P_i V_i}{R T_i}$

$= \frac{100 \times 1000 \times 2}{287 \times 295}$

$= 2.362\ kg$

Final state of tank

$P_f = 600\ kPa$

$V_f = 2\ m^3$

$T_f = 77°C = 350\ K$

$m_{final} = \frac{P_f V_f}{R T_f}$

$= \frac{600 \times 1000 \times 2}{287 \times 350}$

$= 11.946\ kg$

Substituting these quantities in eqn (2),

$m_{in} = m_{final} - m_{initial}$

$= 11.946 - 2.362 = 9.584\ kg$ Ans

To determine (b) amount of heat transfer:-

Look at eqn (4). $\Rightarrow Q_{in} - Q_{out} + h_{in} m_{in} = m_f u_f - m_i u_i$

$\therefore Q_{in} - Q_{out} = (m_f u_f - m_i u_i - m_{in} h_{in})$ — (5)

$m_f = 11.946\ kg$, $u_f = C_v T_f = 0.718\ \frac{kJ}{kg\,K} \times 350\ K$

$= 251.3\ \frac{kJ}{kg}$

$m_i = 2.362\ kg$, $u_i = C_v T_i = 0.718\ \frac{kJ}{kg\,K} \times 295\ K$

$= 211.81\ \frac{kJ}{kg}$

$m_{in} = 9.584\ kg$, $h_{in} = C_p T_{in} = 1.018\ \frac{kJ}{kg\,K} \times 295\ K$

$= 300.31\ \frac{kJ}{kg}$

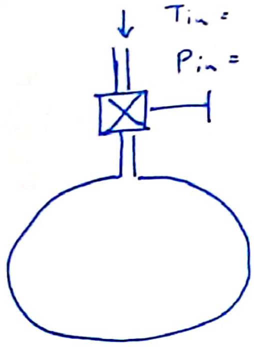
Substituting these values in eqn (5):-

$Q_{in} - Q_{out} = (11.946 \times 251.3) - (2.362 \times 211.81) - (9.584 \times 300.31)$

$= 3002.03\ kJ - 500.295\ kJ - 2878.17\ kJ = 376.436\ kJ$

4)

$\downarrow$ $T_{in} = 25°C = 298K$.
$P_{in} = 125 kPa$.



$C_p(He) = 5.192\ kJ/kg/K$
$C_v(He) = 3.1156\ kJ/kg/K$
$R(He) = 2.0769\ kJ/kg/K$

$\rightarrow$ Balloon initially contains $40 m^3$ of He gas at $100 kPa$ and $17°C = 290K$

The valve is opened, and He is allowed to enter the balloon till pressure equilibrium is reached, i.e., pressure of He inside the balloon is $125 kPa$.

The material of the balloon is such that its volume increases linearly with pressure.

(i) No heat transfer.
(ii) KE & PE interaction ignored.
(iii) $m_{out} = 0$

Mass balance :-

$$m_{in} - \cancel{m_{out}} = m_f - m_i$$

$$\Rightarrow m_{in} = m_f - m_i \quad — (1).$$

Energy balance :-

$$\cancel{Q_{in}} + W_{in} + m_{in}\left[h_{in} + \cancel{KE_{in}} + \cancel{PE_{in}}\right] - \left[\cancel{Q_{out}} + W_{out} + \cancel{m_{out}}\left\{h_{out} + KE_{out} + PE_{out}\right\}\right]$$

$$= m_f\left[u_f + \cancel{KE_f} + \cancel{PE_f}\right] - m_i\left[u_i + \cancel{KE_i} + \cancel{PE_i}\right]$$

$$\Rightarrow W_{in} + m_{in}h_{in} - W_{out} = m_f u_f - m_i u_i$$

$$\Rightarrow W_{in} - W_{out} = \left(m_f u_f - m_i u_i - m_{in}h_{in}\right) \quad — (2).$$

$W_{in} - W_{out} = (m_f u_f - m_i u_i - m_{in} h_{in})$ — (2)

m_i = mass of ~~air~~ He in balloon initially

$$= \frac{P_i V_i}{R T_i} = \frac{100 \times 40}{2.0764 \times 290} = 6.643 \text{ kg} \quad - (3)$$

Since the volume increases linearly with pressure,

$$\frac{P_i}{P_f} = \frac{V_i}{V_f} \Rightarrow \frac{100}{125} = \frac{40 m^3}{V_f} \Rightarrow V_f = 50 m^3 \quad - (4)$$

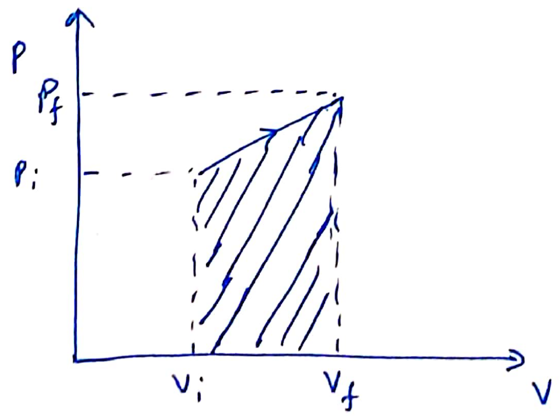
$$m_f = \frac{P_f V_f}{R T_f} = \frac{125 \times 50}{2.0764 \times T_f} = \frac{3010}{T_f} \text{ kg} \quad - (5).$$

$(V_f = 50 m^3 \text{ from } (4))$

Thus, from (1), $m_{in} = m_f - m_i$

$$= \left(\frac{3010}{T_f} - 6.643\right) \text{ kg} \quad - (6)$$

Also, since volume increases linearly with pressure,



Boundary work done during this process is given by the shaded area (area under the curve).

$$\therefore W_{in} - W_{out} = \frac{(P_i + P_f)}{2} \times (V_f - V_i)$$

$$= \frac{(100 + 125)\,kPa}{2} \times (50 - 40)\,m^3$$

$$= 1125\,kJ \quad - \quad (7)$$

Substituting (3), (5), (6) & (7) in eqn (2),

$$1125 = \left(\frac{3010}{T_f} \times C_v \times T_f\right) - (6.643 \times C_v \times T_i) - \left[\left(\frac{3010}{T_f} - 6.643\right) \times C_p \times T_{in}\right]$$

$$\cancel{1125 = (3010 \times C_v)}$$

$$1125 = \cancel{300}\,(3010 \times 3.1156) - (6.643 \times 3.1156 \times 290) - \left[\left(\frac{3010}{T_f} - 6.643\right) \times 5.192 \times 298\right]$$

$$\Rightarrow 1125 = 9377.956 - 6002.1 - \left[\left(\frac{3010}{T_f} - 6.643\right) \times 1547.2\right]$$

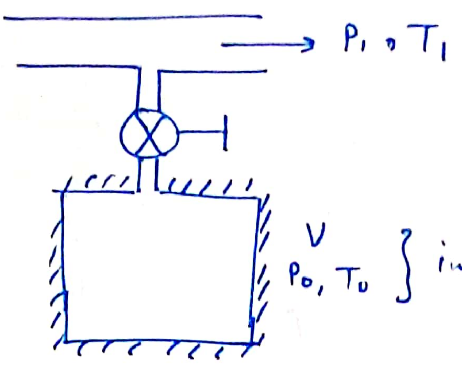
$$\Rightarrow \frac{3010}{T_f} - 6.643 = \frac{9377.956 - 6002.1 - 1125}{1547.2}$$

$$= 1.4547$$

$$\Rightarrow \frac{3010}{T_f} = 1.4547 + 6.643 = 8.0977$$

$$\Rightarrow T_f = \frac{3010}{8.0977} = 371.71\,K.$$

5.)



$\left.\begin{matrix} V \\ P_0, T_0 \end{matrix}\right\}$ initial conditions of cylinder.

The valve is opened, and the air from the pressure line fills the cylinder till the cylinder pressure reaches P_1.

(i) The cylinder is rigid & insulated. Thus, $Q_{in} = Q_{out} = 0$
and $W_{in} = W_{out} = 0$

(ii) No mass flows out of the cylinder, so, $m_{out} = 0$

(iii) KE & PE interactions are ignored.

<u>Mass balance</u> :-

$$m_{in} - \cancel{m_{out}} = m_f - m_i$$

$$\Rightarrow \quad m_{in} = m_f - m_i \quad — (1)$$

<u>Energy balance</u> :-

$$\cancel{Q_{in}} + \cancel{W_{in}} + m_{in}\left[h_{in} + \cancel{KE_{in}} + \cancel{PE_{in}}\right] - \left[\cancel{Q_{out}} + \cancel{W_{out}} + \cancel{m_{out}}\left\{h_{out} + KE_{out} + PE_{out}\right\}\right]$$

$$= m_f\left[u_f + \cancel{KE_f} + \cancel{PE_f}\right] - m_i\left[u_i + \cancel{KE_i} + \cancel{PE_i}\right]$$

$$\Rightarrow \quad m_{in} h_{in} = m_f u_f - m_i u_i \quad — (2)$$

From initial conditions, $m_i = \dfrac{P_0 V}{R T_0}$ — (3)

From final conditions, $m_f = \dfrac{P_1 V}{R T_f}$ — (4) $\left[T_f \text{ unknown}\right]$.

Substitute (3) & (4) in (1),

$$m_{in} = m_f - m_i = \frac{P_1 V}{R T_f} - \frac{P_o V}{R T_o}$$

$$= \frac{V}{R}\left[\frac{P_1}{T_f} - \frac{P_o}{T_o}\right] \quad -(5)$$

Substitute (5), (3) & (4) in (2),

Eqn (2) :- $m_{in} h_{in} = m_f u_f - m_i u_i$

$$\Rightarrow \frac{V}{R}\left[\frac{P_1}{T_f} - \frac{P_o}{T_o}\right] C_p T_1 = \frac{P_1 V}{R T_f} \cdot C_v T_f - \frac{P_o V}{R T_o} \cdot C_v T_o \quad -(6)$$

[Since $h_{in} = C_p T_1$,
$u_f = C_v T_f$,
$u_i = C_v T_o$].

⇒ Cancelling $\frac{V}{R}$ from both sides of eqn (6),

$$\left(\frac{P_1}{T_f} - \frac{P_o}{T_o}\right) C_p T_1 = \frac{P_1}{\cancel{T_f}} \cdot C_v \cancel{T_f} - \frac{P_o}{\cancel{T_o}} \cdot C_v \cancel{T_o}$$

$$\Rightarrow \left(\frac{P_1}{T_f} - \frac{P_o}{T_o}\right) C_p T_1 = C_v \left[P_1 - P_o\right]$$

$$\Rightarrow \frac{P_1}{T_f} - \frac{P_o}{T_o} = \frac{P_1 - P_o}{T_1 \gamma} \qquad \left[\frac{C_v}{C_p} = \frac{1}{\gamma}\right]$$

$$\frac{P_1}{T_f} = \frac{P_0}{T_0} + \frac{P_1}{\gamma T_1} - \frac{P_0}{\gamma T_1}$$

$\Rightarrow$

$$\frac{1}{T_f} = \frac{P_0}{P_1} \cdot \frac{1}{T_0} + \frac{1}{\gamma T_1} - \frac{P_0}{P_1} \cdot \frac{1}{\gamma T_1}$$

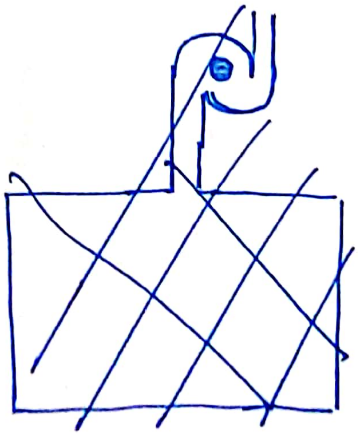
$$= \frac{P_0}{P_1} \cdot \frac{\gamma T_1}{\gamma T_0 T_1} + \frac{1}{\gamma T_1} - \frac{P_0}{P_1} \cdot \frac{1}{\gamma T_1}$$

$$= \frac{1}{\gamma T_1} \left[\frac{P_0}{P_1} \cdot \gamma \frac{T_1}{T_0} + 1 - \frac{P_0}{P_1} \right]$$

$$= \frac{1}{\gamma T_1} \left[\frac{P_0}{P_1} \left\{ \gamma \frac{T_1}{T_0} - 1 \right\} + 1 \right]$$

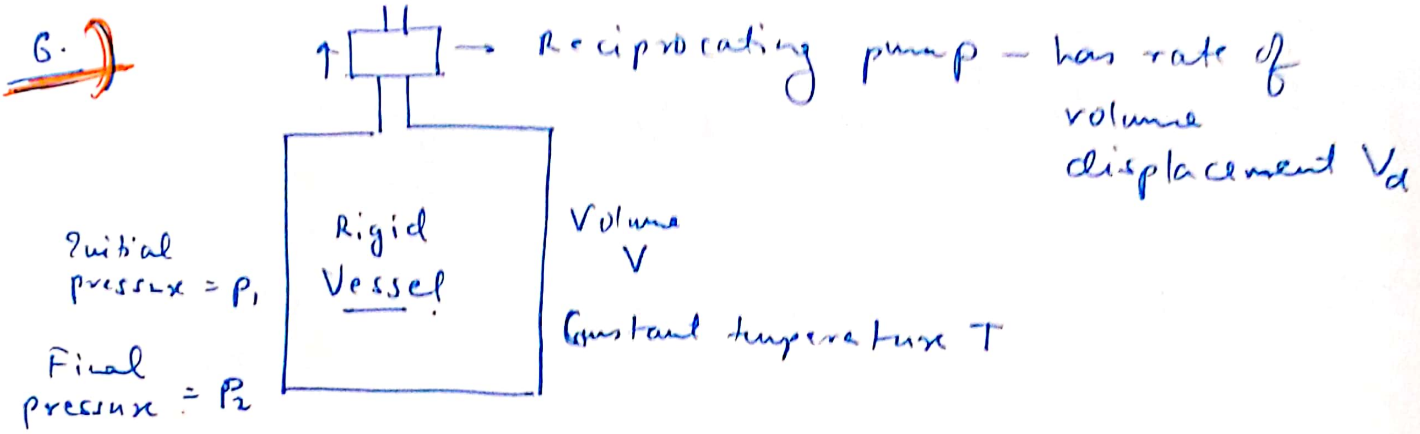
$$\therefore \quad T_f = \frac{\gamma T_1}{1 + \frac{P_0}{P_1}\left(\gamma \frac{T_1}{T_0} - 1 \right)}$$

6/ X



P. T. O.

6.)



The vessel has a volume V, and is maintained at a constant temperature T by energy transfer as heat.

Initial pressure in the vessel = p_1

Final pressure in the vessel = p_2

A reciprocating pump having rate of volume displacement V_d is used to ~~or~~ take air out of the vessel.

Aim :- To calculate :- (a) time taken from pressure in the vessel to drop from P_1 to P_2

(b) necessary heat transfer as heat during evacuation.

$pV = mRT$ valid – (1)

From the ideal gas equation $pV = mRT$, differentiating both sides,

$$p\,dV + V\,dp = mR\,dT + dm \cdot RT \quad - (2)$$

In equation (2);

since temperature of air is constant, $dT = 0$

since the vessel is rigid, $dV = 0$

$\therefore$ eqn (2) reduces to :-

$$Vdp = dm\, RT$$

$$\Rightarrow \quad dm = \frac{Vdp}{RT} \quad - (3)$$

Now, the pump extracts air equivalent to V_d volume per unit time, from the vessel.

Thus, the mass of air that the pump extracts from the vessel per unit time is given by

$$\rho V_d = \frac{p}{RT} V_d.$$

$$\therefore \frac{dm}{dt} = -\frac{p}{RT} V_d$$

[-ve sign because the mass of air inside the vessel is decreasing]

$$\Rightarrow \quad dm = -\frac{p}{RT} V_d\, dt \quad - (4)$$

Equate eqn (3) & eqn (4),

$$\frac{V\,dp}{\cancel{RT}} = -\frac{p}{\cancel{RT}} V_d\, dt$$

$$\Rightarrow V dp = -p V_d\, dt \Rightarrow \frac{dp}{p} = -\frac{V_d}{V} dt$$

P.T.O.

$$\frac{dp}{p} = -\frac{V_d}{V}\,dt.$$

Integrate both sides,

$$\int \frac{dp}{p} = -\frac{V_d}{V}\int dt.$$

At $t = 0$, $p = p_1$

At final time t, $p = p_2$

$$\therefore \int_{p_1}^{p_2} \frac{dp}{p} = -\frac{V_d}{V}\int_0^t dt$$

$$\therefore \ln\left(\frac{p_2}{p_1}\right) = -\frac{V_d}{V}t$$

$$\Rightarrow \ln\left(\frac{p_1}{p_2}\right) = \frac{V_d}{V}t \Rightarrow t = \frac{V}{V_d}\ln\left(\frac{p_1}{p_2}\right)$$

— Ans

To calculate necessary ~~heat~~ energy transfer as heat :-

Energy balance :-

$$Q_{in} + W_{in} + m_{in}\left[h_{in} + KE_{in} + PE_{in}\right] -$$

$$\left[Q_{out} + W_{out} + m_{out}\left\{h_{out} + KE_{out} + PE_{out}\right\}\right] =$$

$$m_f\left[u_f + KE_f + PE_f\right] - m_i\left[u_i + KE_i + PE_i\right] \quad -(5)$$

In eqn (5),

(i) KE & PE interactions are ignored.

(ii) Since the vessel is rigid, $W_{in} = W_{out} = 0$

(iii) Since no mass enters the vessel, $m_{in} = 0$

$\therefore$ Eqn (5) reduces to;

$$(Q_{in} - Q_{out}) - m_{out} h_{out} = m_f u_f - m_i u_i \quad - (6)$$

Mass balance:-

$$m_{in} - m_{out} = \cancel{m_f - u_f} \; m_f - m_i$$

Since $m_{in} = 0$;

$$m_{out} = m_i - m_f \quad - (7)$$

Now, $m_i = \dfrac{P_1 V}{RT}$ and $m_f = \dfrac{P_2 V}{RT}$ $\quad - (8)$

Substitute (8) in (7),

$$m_{out} = m_i - m_f = \frac{P_1 V}{RT} - \frac{P_2 V}{RT} = \frac{V}{RT}(P_1 - P_2) \quad - (9)$$

Substitute (8) and (9) in (6),

$$(Q_{in} - Q_{out}) - \frac{V}{RT}(P_1 - P_2) h_{out} = \frac{P_2 V}{RT} u_f - \frac{P_1 V}{RT} u_i \quad - (10)$$

Now,
$$\left.\begin{aligned} h_{out} &= C_p T \\ u_f &= C_v T \\ u_i &= \cancel{T}\, C_v T \end{aligned}\right] - (11)$$

Substitute (11) in (10),

$$(Q_{in} - Q_{out}) - \frac{V}{RT}(P_1 - P_2) \cdot C_p T = \frac{P_2 V}{RT} C_v T - \frac{P_1 V}{RT} \cdot C_v T$$

$$\Rightarrow (Q_{in} - Q_{out}) - \frac{V}{R} C_p (P_1 - P_2) = \frac{V C_v}{R}(P_2 - P_1)$$

$$\Rightarrow Q_{in} - Q_{out} = \frac{V C_v}{R}(P_2 - P_1) + \frac{V C_p}{R}(P_1 - P_2)$$

$$= \frac{-V C_v}{R}(P_1 - P_2) + \frac{V C_p}{R}(P_1 - P_2)$$

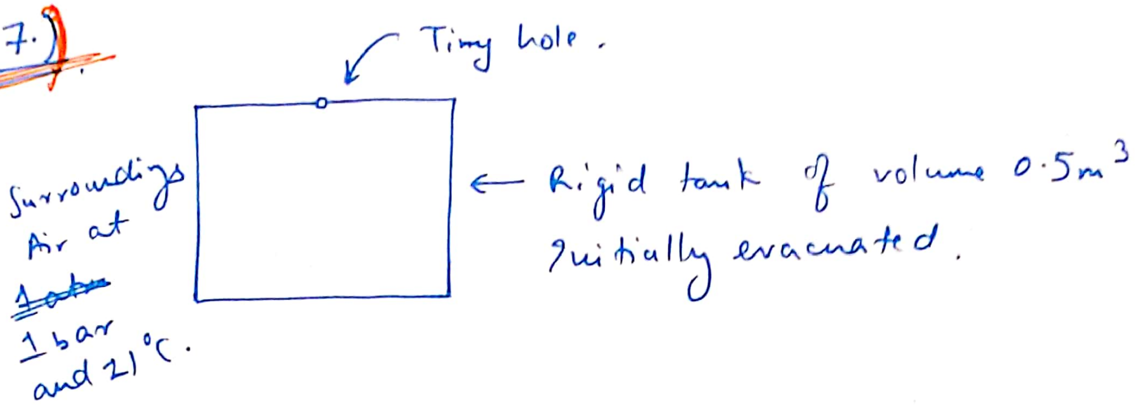
$$= \frac{V(P_1 - P_2)}{R}\left[-C_v + C_p\right]$$

$$= \frac{V}{R}(P_1 - P_2) \cdot R$$

$$= (P_1 - P_2)V$$

$\therefore$ Heat transfer $= \underline{\underline{V(P_1 - P_2)}}$ Ans

7.)



Initially, the rigid tank is evacuated. The surroundings has air at 1 bar and 21 °C.

A hole develops in the wall, and air starts to leak in until the pressure inside the tank reaches 1 bar. The temperature of the air inside the tank remains constant at 21°C.

$C_p = 1.018$ kJ/kg/K
$C_v = 0.718$ kJ/kg/K
$R = 0.287$ kJ/kg/K.

<u>Mass balance</u> :-

$$m_{in} - m_{out} = m_f - m_i \quad — ①$$

(i) No mass flows out of the tank, so $m_{out} = 0$

(ii) Initially the tank is evacuated, so $m_i = 0$

So, eqn ① reduces to :-

$$m_{in} = m_f \quad — ②$$

Final state of tank :- $V = 0.5 m^3$

$P = 1$ bar $= 10^5$ Pa

$T = 21°C = 294$ K

$$\therefore m_f = \frac{10^5 \times 0.5}{287 \times 294} = 0.592 \text{ kg} = m_{in} \quad ③$$

<u>P.T.O.</u>

Energy balance :-

$$Q_{in} + W_{in} + m_{in}\left[h_{in} + KE_{in} + PE_{in}\right] - \left[Q_{out} + W_{out} + m_{out}\left\{h_{out} + KE_{out} + PE_{out}\right\}\right] =$$

$$m_f\left[u_f + KE_f + PE_f\right] - m_i\left[u_i + KE_i + PE_i\right] \quad -(4)$$

In eqn (4);

(i) Rigid tank, so $W_{in} = W_{out} = 0$

(ii) $m_{out} = 0$

(iii) $m_i = 0$ as the tank is initially evacuated.

(iv) KE & PE interactions are neglected.

$\therefore$ Eqn (4) reduces to :-

$$Q_{in} + m_{in}h_{in} - Q_{out} = m_f u_f.$$

$$\Rightarrow Q_{in} - Q_{out} = m_f u_f - m_{in}h_{in} \quad -(5)$$

From (2), $m_f = m_{in}$

$$\therefore Q_{in} - Q_{out} = m_{in}\left[u_f - h_{in}\right]$$

$$= m_{in}\left[C_v T_f - C_p T_{in}\right]$$

According to question, $T_f = T_{in} = 21^\circ C = 294 K$

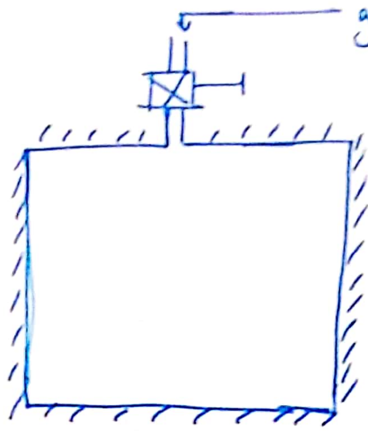
And, according to (3), $m_{in} = 0.592\,kg$.

$$\therefore Q_{in} - Q_{out} = 0.592 \times \left[(0.718 \times 294) - (1.018 \times 294)\right] kJ$$

$$= 0.592 \times 294\left[0.718 - 1.018\right] kJ$$

$$= -52.2144\,kJ \quad \text{Ans.}$$

8.)



Gas from the supply line is at T_1 temperature. It is pumped into the vessel and the inflow varies as:-

$$\dot{m}(t) = \dot{m}_0 e^{-at} \quad - (1)$$

Also, $\underbrace{Pv}_{\text{specific volume}} = RT \Rightarrow \underbrace{PV}_{\text{Total volume.}} = mRT - (2)$

To determine :- pressure & temperature of gas in the vessel as a function of time.

<u>Energy balance</u> :-

$$Q_{in} + W_{in} + m_{in}\left[h_{in} + KE_{in} + PE_{in}\right] -$$

$$\left[Q_{out} + W_{out} + m_{out}\left\{h_{out} + KE_{out} + PE_{out}\right\}\right]$$

$$= m_f\left[u_f + KE_f + PE_f\right] - m_i\left[u_i + KE_i + PE_i\right] \quad - (3)$$

In eqn (3):-

(i) Vessel is rigid, so $W_{in} = W_{out} = 0$
(ii) Vessel is insulated, so $Q_{in} = Q_{out} = 0$
(iii) KE & PE interactions are ignored.
(iv) No mass flows out of the vessel, so $m_{out} = 0$.

(4)

~~Substitute~~

Applying conditions in (4) to eqn (3) :-

$$\cancel{Q_{in}} + \cancel{W_{in}} + m_{in}\left[h_{in} + \cancel{KE_{in}} + \cancel{PE_{in}}\right] -$$

$$\left[\cancel{Q_{out}} + \cancel{W_{out}} + \cancel{m_{out}}\left\{h_{out} + KE_{out} + PE_{out}\right\}\right]$$

$$= m_f\left[u_f + \cancel{KE_f} + \cancel{PE_f}\right] - m_i\left[u_i + \cancel{KE_i} + \cancel{PE_i}\right]$$

$$\Rightarrow \quad m_{in} h_{in} = m_f u_f - m_i u_i$$

$$= E_f - E_i \quad \text{— (5)}$$

, E = total internal energy of the system (gas in the vessel)

Since we have mass flow rate given in the question, we convert eqn (5) to a "rate-equation" by ~~taking~~ dividing both sides by Δt.

$$\therefore \quad \frac{m_{in}}{\Delta t} h_{in} = \frac{E_f - E_i}{\Delta t}$$

At $\Delta t \to 0$,

$$\frac{dm_{in}}{dt} h_{in} = \frac{dE_v}{dt} \quad \text{— (6)}$$

, E_v = total internal energy of the gas in the vessel.

$$\frac{dm_{in}}{dt} = \dot{m}_{in} = \dot{m}_0 e^{-at} \quad \text{— (7)}$$

Substitute (7) in (6),

$$\frac{dE_v}{dt} = h_{in} \dot{m}_{in} = h_{in} \dot{m}_o e^{-at}$$

$$dE_v = h_{in} \dot{m}_o e^{-at} dt$$

Integrating both sides,

$$\int_{E_0}^{E} dE_v = h_{in} \dot{m}_o \int_0^t e^{-at} dt$$

$$= \frac{h_{in} \dot{m}_o}{-a} e^{-at} \Big|_0^t$$

$$= \frac{h_{in} \dot{m}_o}{-a} \left[e^{-at} - 1 \right]$$

$$E - E_0 = \frac{h_{in} \dot{m}_o}{a} \left(1 - e^{-at} \right)$$

$$\Rightarrow \quad E = E_0 + \frac{h_{in} \dot{m}_o}{a} \left(1 - e^{-at} \right) \quad - (8)$$

Now, $E = Mu$

$E_0 = M_0 u_0 \rightarrow$ at initial conditions.

$$\therefore \quad Mu = M_0 u_0 + \frac{h_{in} \dot{m}_o}{a} \left(1 - e^{-at} \right)$$

$$= M_0 u_0 + \frac{\dot{m}_o}{a} \left(u_{in} + P v_{in} \right) \left(1 - e^{-at} \right)$$

{ Because $h_{in} = u_{in} + P v_{in}$ }

P.T.O.

$$Mu = M_0 u_0 + \frac{\dot{m}_0}{a}(u_{in} + Pv_{in})(1 - e^{-at})$$

$$= M_0 u_0 + \frac{\dot{m}_0}{a}(u_{in} + RT_{in})(1 - e^{-at})$$

$\left\{ \text{Because } Pv = RT \right\}$

$$\Rightarrow Mu = M_0 u_0 + \frac{\dot{m}_0}{a}(u_{in} + RT_1)(1 - e^{-at}) \quad - \ 9$$

$\left(\text{Because } T_{in} = T_1 \right)$

Also, lets consider mass flow rate equation.

$$\frac{dm}{dt} = \dot{m}_0 e^{-at}$$

$$\Rightarrow dm = \dot{m}_0 e^{-at} dt$$

Integrate both sides,

$$\int_{M_0 = \text{initial mass}}^{M} dm = \dot{m}_0 \int_0^t e^{-at} dt$$

$$\Rightarrow M - M_0 = \frac{\dot{m}_0}{-a}\left[e^{-at}\right]\Big|_0^t = \frac{\dot{m}_0}{a}(1 - e^{-at})$$

$$\therefore M = M_0 + \frac{\dot{m}_0}{a}(1 - e^{-at}) \quad - \ 10$$

Now, lets eliminate 'M' from equations (9) and (10).

Substitute the expression of M from eqn (10) into LHS of eqn (9).

$$\left[M_0 + \frac{\dot{m}_0}{a}\left(1-e^{-at}\right)\right]u = M_0 u_0 + \frac{\dot{m}_0}{a}\left(u_{in} + RT_1\right)\left(1-e^{-at}\right)$$

$\Rightarrow$

$$M_0 u - M_0 u_0 = \frac{\dot{m}_0}{a}\left(u_{in} + RT_1\right)\left(1-e^{-at}\right) - \frac{\dot{m}_0}{a}\left(1-e^{-at}\right)u$$

$\Rightarrow$

$$M_0 C_v \left(T - T_0\right) = \frac{\dot{m}_0}{a}\left(1-e^{-at}\right)\left[u_{in} + RT_1 - u\right]$$

$$= \frac{\dot{m}_0}{a}\left(1-e^{-at}\right)\left[C_v T_1 + RT_1 - u\right]$$

$$\left\{ \begin{aligned} u_{in} &= C_v T_{in} \\ &= C_v T_1 \end{aligned} \right\}.$$

$$\cancel{= \frac{\dot{m}_0}{a}\left(1-e^{-at}\right)\left[\left(C_v + R\right)T_1 -\right.}$$

$$= \frac{\dot{m}_0}{a}\left(1-e^{-at}\right)\left[C_v T_1 + RT_1 - C_v T\right]$$

$$\Rightarrow M_0 C_v T - M_0 C_v T_0 = \frac{\dot{m}_0}{a}\left(1-e^{-at}\right)\left[\left(C_v + R\right)T_1\right] - \frac{\dot{m}_0}{a}\left(1-e^{-at}\right)C_v T$$

$$\Rightarrow M_0 C_v T + \frac{\dot{m}_0}{a}\left(1-e^{-at}\right)C_v T = \frac{\dot{m}_0}{a}\left(1-e^{-at}\right)C_p T_1 + M_0 C_v T_0$$

P.T.O

$$\left[C_v + R = C_p\right].$$

From last page,

$$M_0 C_v T + \frac{\dot{m}_0}{a}(1-e^{-at}) C_v T = \frac{\dot{m}_0}{a}(1-e^{-at}) C_p T_1 + M_0 C_v T_0$$

$\Rightarrow$

$$T\left[M_0 C_v + \frac{\dot{m}_0}{a}(1-e^{-at}) C_v\right] = \frac{\dot{m}_0}{a}(1-e^{-at}) C_p T_1 + M_0 C_v T_0$$

$$\Rightarrow T = \frac{\frac{\dot{m}_0}{a}(1-e^{-at}) C_p T_1 + M_0 C_v T_0}{\left[M_0 + \frac{\dot{m}_0}{a}(1-e^{-at})\right] C_v} \quad \text{— (11)}$$

Ans

— This is the variation of gas temperature w.r.t. time.

From eqn (10), $M = M_0 + \frac{\dot{m}_0}{a}(1-e^{-at})$. This term also appears in the denominator for expression of temperature in eqn (11).

$\therefore$ If we substitute eqn (10) in eqn (11),

$$T = \frac{\frac{\dot{m}_0}{a}(1-e^{-at}) C_p T_1 + M_0 C_v T_0}{M C_v}$$

$$\Rightarrow MT = \frac{1}{C_v}\left[\frac{\dot{m}_0}{a}(1-e^{-at}) C_p T_1 + M_0 C_v T_0\right] \quad \text{— (12)}$$

For pressure variation, we know that

$$PV = MRT \quad - \text{eqn } (2)$$

$$\therefore \frac{(MT) \cdot R}{V} = P \quad - (13)$$

Substitute the value of (MT) from eqn (12) into eqn (13).

$$P = \frac{R}{V C_v}\left[\frac{\dot{m}_0}{a}\left(1-e^{-at}\right) C_p T_1 + M_0 C_v T_0\right]$$

$$= \frac{R}{V C_v} \cdot \frac{\dot{m}_0}{a} C_p T_1 \left(1-e^{-at}\right) + \frac{R}{V \cancel{C_v}} \cdot M_0 \cancel{C_v} T_0$$

$$= \frac{\dot{m}_0}{a} \cdot \frac{R}{V} \cdot \gamma T_1 \left(1-e^{-at}\right) + \frac{R M_0 T_0}{V} \quad - (14)$$

The last expression on RHS of eqn (14),

$$\frac{R M_0 T_0}{V} = P_0 = \text{initial pressure in the vessel.}$$

$$\therefore P = P_0 + \frac{\dot{m}_0}{a} \frac{R}{V} \gamma T_1 \left(1-e^{-at}\right) \quad - (15)$$

Ans.

- This is the gas pressure variation w.r.t. ~~ga~~ time (of the gas inside the vessel).

∴ To summarize,

$$T = \frac{M_0 C_v T_0 + \frac{\dot{m}_0}{a}(1-e^{-at}) C_p T_1}{\left[M_0 + \frac{\dot{m}_0}{a}(1-e^{-at})\right] C_v} \quad - \quad \text{eqn (11)}$$

$$P = P_0 + \frac{\dot{m}_0}{a} \frac{R}{V} \cdot \gamma T_1 (1-e^{-at}) \quad - \quad \text{eqn (15)}$$

(a) If the vessel was initially evacuated, $M_0 = 0$.

∴ In eqn (11),

$$T = \frac{\frac{\dot{m}_0}{a}(1-e^{-at}) C_p T_1}{\frac{\dot{m}_0}{a}(1-e^{-at}) C_v} = \gamma T_1$$

∴ If the vessel was initially evacuated,

$T(t) = \gamma T_1$ = Constant.

Thus, the temperature inside the vessel is independent of time.

(b) To determine the charging time when the pressure inside the vessel reaches that of supply main.

At this point, $P = P_1$ in eqn (15).

(15) :-

$$P_1 - P_0 = \frac{\dot{m}_0}{a} \cdot \frac{R}{V} \gamma T_1 \left(1 - e^{-at_{P_1}}\right) ;$$

t_{P_1} = charging time to reach supply main pressure.

$$\Rightarrow \quad \frac{P_1 - P_0}{\frac{\dot{m}_0}{a} \cdot \frac{R}{V} \cdot \gamma T_1} = 1 - e^{-at_{P_1}}$$

$$\Rightarrow \quad \frac{(P_1 - P_0)\, aV}{\dot{m}_0 R \gamma T_1} = 1 - e^{-at_{P_1}}$$

$$\Rightarrow \quad e^{-at_{P_1}} = 1 - \frac{(P_1 - P_0)\, aV}{\dot{m}_0 R \gamma T_1}$$

Taking $\log_e$ on both sides,

$$\ln\left(e^{-at_{P_1}}\right) = \ln\left[1 - \frac{(P_1 - P_0)\, aV}{\dot{m}_0 R \gamma T_1}\right]$$

$$\Rightarrow \quad -at_{P_1} = \ln\left[1 - \frac{(P_1 - P_0)\, aV}{\dot{m}_0 R \gamma T_1}\right] = \ln\left[\frac{\dot{m}_0 R \gamma T_1 - (P_1 - P_0)\, aV}{\dot{m}_0 R \gamma T_1}\right]$$

$\cancel{\Rightarrow t = -\frac{1}{a} \ln}$

$$\Rightarrow \quad at_{P_1} = \ln\left[\frac{\dot{m}_0 R \gamma T_1}{\dot{m}_0 R \gamma T_1 - (P_1 - P_0)\, aV}\right]$$

$$\Rightarrow \quad t_{P_1} = \frac{1}{a}\left[\frac{\dot{m}_0 R \gamma T_1}{\dot{m}_0 R \gamma T_1 - (P_1 - P_0)\, aV}\right] \quad \underline{\text{Ans}}.$$